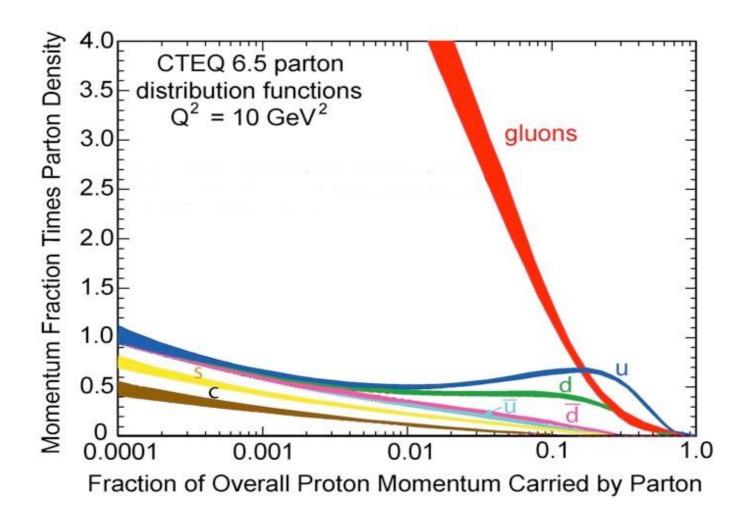
### TMDs at small x

Daniël Boer POETIC 2016, November 16, 2016



#### Partons at small x

Gluons dominate at high center of mass energy s, where the gluons carry a small fraction of the proton momentum:  $x \approx Q^2/s \ll I$ 



At small x it becomes natural to consider the transverse momentum dependence

TMD = transverse momentum dependent parton distribution

Because of the additional  $k_T$  dependence there are more TMDs than collinear pdfs

#### Gluons TMDs

#### The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \text{F.T.} \langle P|\text{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

#### For unpolarized protons:

$$\Gamma_{U}^{\mu\nu}(x, \mathbf{p}_{T}) = \frac{x}{2} \left\{ -g_{T}^{\mu\nu} (f_{1}^{g}(x, \mathbf{p}_{T}^{2}) + \left(\frac{p_{T}^{\mu}p_{T}^{\nu}}{M_{p}^{2}} + g_{T}^{\mu\nu} \frac{\mathbf{p}_{T}^{2}}{2M_{p}^{2}}\right) h_{1}^{\perp g}(x, \mathbf{p}_{T}^{2}) \right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

Gluons inside unpolarized protons can be polarized!

#### For transversely polarized protons:

gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2} \left\{ g_T^{\mu\nu} \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} \left( f_{1T}^{\perp g}(x,\boldsymbol{p}_T^2) + \dots \right) \right\}$$

## Process dependence

#### Process dependence of gluon TMDs

$$\Gamma_g^{\mu\nu}(x,k_T) \equiv \text{F.T.}\langle P|\text{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

The gauge links are process dependent, affecting even the unpolarized gluon TMDs as was first realized in a small-x context

Dominguez, Marquet, Xiao, Yuan, 2011

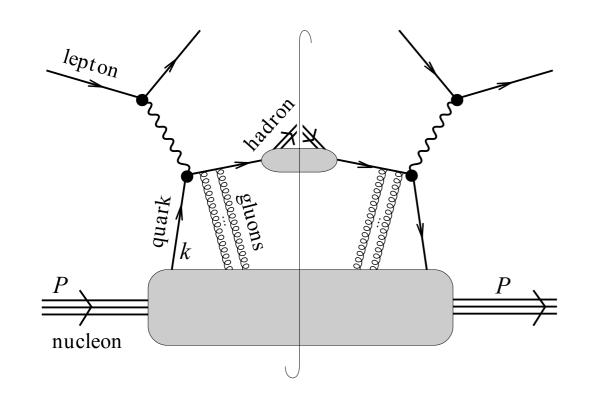
Kharzeev, Kovchegov & Tuchin (2003): ``A tale of two gluon distributions'' They noted there are 2 distinct but equally valid definitions for the small-x gluon distribution: the Weizsäcker-Williams (WW) and the dipole (DP) distribution

KKT: "cannot offer any simple physical explanation of this paradox"

The explanation turns out to be in the process dependence of the gluon distribution, in other words, its sensitivity to the initial and/or final state interactions (ISI/FSI) in a process

WW and DP distributions would be the same without ISI/FSI

#### Initial and final state interactions



summation of all gluon rescatterings leads to path-ordered exponentials in correlators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P} \exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_{\mu} A^{\mu}(s)\right)$$

$$\Phi \propto \langle P | \overline{\psi}(0) \mathcal{L}_{\mathcal{C}}[0, \xi] \psi(\xi) | P \rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

#### Resulting Wilson lines depend on whether the color is incoming or outgoing

[Collins & Soper, 1983; D.B. & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, X. Ji & F.Yuan, 2003; D.B., Mulders & Pijlman, 2003]

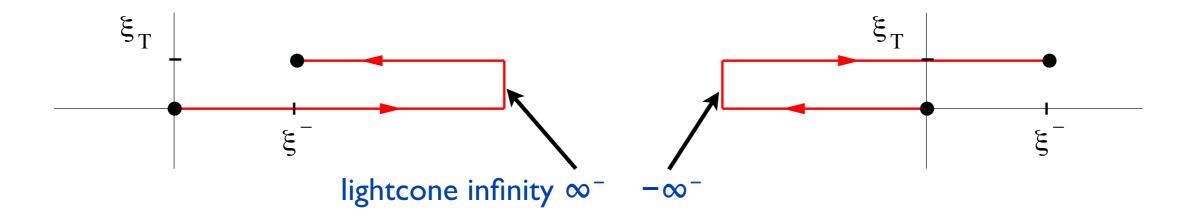
This does not automatically imply that the ISI and/or FSI affect observables, but it turns out that they do in certain cases, for example, Sivers effect asymmetries [Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

#### Process dependence of Sivers TMDs

#### SIDIS

FSI lead to a future pointing Wilson line (+ link), whereas ISI to past pointing (- link)

DY



One can use parity and time reversal invariance to relate these

$$f_{1T}^{\perp q {
m [SIDIS]}}(x,k_T^2) = -f_{1T}^{\perp q {
m [DY]}}(x,k_T^2)$$
 [Collins '02]

A similar sign change relation for gluon Sivers functions holds, but due to the appearance of two gauge links, there are more possibilities

#### Sign change relation for gluon Sivers TMD

$$e \, p^{\uparrow} \to e' \, Q \bar{Q} \, X$$

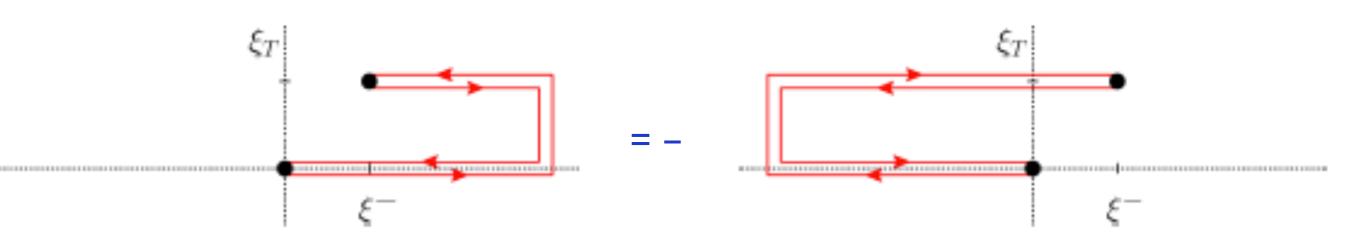
$$\gamma^*\,g o Qar Q$$
 probes [+,+]

$$p^{\uparrow} p \rightarrow \gamma \gamma X$$

Qiu, Schlegel, Vogelsang, 2011

In the kinematic regime where pair rapidity is central, one effectively selects the subprocess:

$$g\,g o \gamma\,\gamma$$
 probes [-,-]



$$f_{1T}^{\perp g \, [e \, p^{\uparrow} \to e' \, Q \, \overline{Q} \, X]}(x, p_T^2) = -f_{1T}^{\perp g \, [p^{\uparrow} \, p \to \gamma \, \gamma \, X]}(x, p_T^2)$$

D.B., Mulders, Pisano, Zhou, 2016

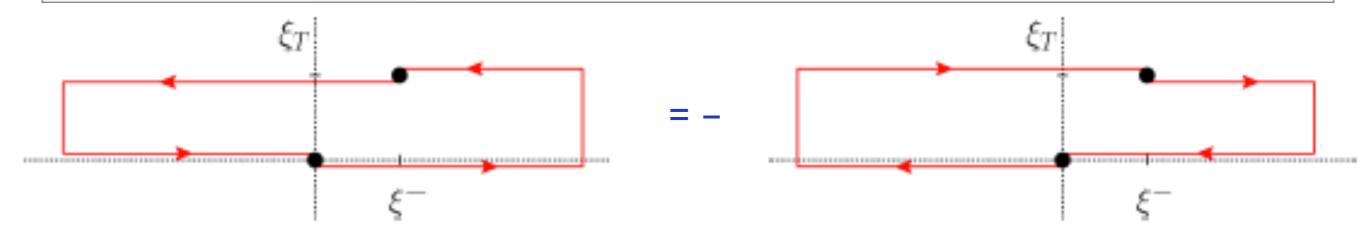
#### f and d type gluon Sivers TMD

$$e \, p^{\uparrow} \to e' \, Q \bar{Q} \, X$$

$$\gamma^*\,g o Qar Q$$
 probes [+,+]

$$p^{\uparrow} p \to \gamma \operatorname{jet} X$$

In the kinematic regime where gluons in the polarized proton dominate, one effectively selects the subprocess:  $g\,q o \gamma\,q$  probes [+,-]



These processes probe 2 distinct, independent gluon Sivers functions

Related to antisymmetric (fabc) and symmetric (dabc) color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Conclusion: gluon Sivers TMD studies at EIC and at RHIC or AFTER@LHC can be related or complementary, depending on the processes considered

# Unpolarized gluon TMDs at small x

#### WW vs DP

For most processes of interest there are 2 relevant unpolarized gluon distributions

Dominguez, Marquet, Xiao, Yuan, 2011

$$xG^{(1)}(x,k_{\perp}) = 2 \int \frac{d\xi^{-}d\xi_{\perp}}{(2\pi)^{3}P^{+}} e^{ixP^{+}\xi^{-}-ik_{\perp}\cdot\xi_{\perp}} \langle P|\text{Tr}\left[F^{+i}(\xi^{-},\xi_{\perp})\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \qquad [+,+]$$

$$xG^{(2)}(x,k_{\perp}) = 2 \int \frac{d\xi^{-}d\xi_{\perp}}{(2\pi)^{3}P^{+}} e^{ixP^{+}\xi^{-}-ik_{\perp}\cdot\xi_{\perp}} \langle P|\text{Tr}\left[F^{+i}(\xi^{-},\xi_{\perp})\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \quad [+,-]$$

For unpolarized gluons [+,+] = [-,-] and [+,-] = [-,+]

At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_{\perp}) = -\frac{2}{\alpha_S} \int \frac{d^2v}{(2\pi)^2} \frac{d^2v'}{(2\pi)^2} e^{-ik_{\perp}\cdot(v-v')} \left\langle \operatorname{Tr}\left[\partial_i U(v)\right] U^{\dagger}(v') \left[\partial_i U(v')\right] U^{\dagger}(v) \right\rangle_{x_g} \quad \text{WW}$$

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2 \alpha_s} S_\perp \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp \cdot r_\perp} \frac{1}{N_c} \left\langle {\rm Tr} U(0) U^\dagger(r_\perp) \right\rangle_{x_g} \label{eq:equation:e$$

Different processes probe one or the other or a mixture, so this can be tested

#### WW vs DP

#### Selection of processes that probe the WW or DP unpolarized gluon TMD:

	DIS	DY	SIDIS		$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	l .	
$f_1^{g[+,+]} \text{ (WW)}$	×	×	×	×		$\sqrt{}$	
$f_1^{g[+,-]} (DP)$					×	×	×

Dijet production in pA probes a combination of 6 distinct unpolarized gluon TMDs In the large  $N_c$  limit it probes a combination of DP and WW functions

Akcakaya, Schäfer, Zhou, 2013; Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren, 2015

Dijet production in pA generally suffers from factorization breaking contributions

Collins, Qiu, 2007; Rogers, Mulders, 2010

In Y+ $\gamma$  production the color singlet contribution dominates and in J/ $\psi$ + $\gamma$  production for a specific range of invariant mass of the pair

den Dunnen, Lansberg, Pisano, Schlegel, 2014

#### MV model

In the MV model one may not notice the origin for the difference between WW and DP, because the two TMDs become related:

$$xG_g^{(2)}(x,q_{\perp}) \stackrel{\mathsf{MV}}{\propto} q_{\perp}^2 \nabla_{q_{\perp}}^2 xG_g^{(1)}(x,q_{\perp})$$

Processes involving  $G^{(1)}$  (WW) [+,+] in the MV model can be expressed in terms of  $G^{(2)} \sim C(k_{\perp})$ , e.g.

$$\gamma A \rightarrow Q \bar{Q} X \qquad \frac{\mathrm{d}\sigma_{\mathrm{T}}}{\mathrm{d}y \, \mathrm{d}k_{\perp}} = \pi R^2 \frac{2N_{\mathrm{c}}(Z\alpha)^2}{3\pi^3} \ln\left(\frac{\gamma}{2mR}\right) k_{\perp} C(k_{\perp})$$
 Gelis, Peshier, 2002 
$$\times \left\{1 + \frac{4(k_{\perp}^2 - m^2)}{k_{\perp} \sqrt{k_{\perp}^2 + 4m^2}} \operatorname{arcth} \frac{k_{\perp}}{\sqrt{k_{\perp}^2 + 4m^2}} \right\}$$

$$C(\mathbf{k}_{\perp}) = \int d^2 \mathbf{x}_{\perp} \, \mathrm{e}^{\mathrm{i}\mathbf{k}_{\perp} \cdot \mathbf{x}_{\perp}} \langle U(0) U^{\dagger}(\mathbf{x}_{\perp}) \rangle$$

Heavy quark pair production in DIS probes the WW distribution, like  $pp \rightarrow Higgs X$ 

For general x expressions, see Pisano, D.B., Brodsky, Buffing, Mulders, 2013

# Linearly polarized gluons in unpolarized hadrons at small *x*

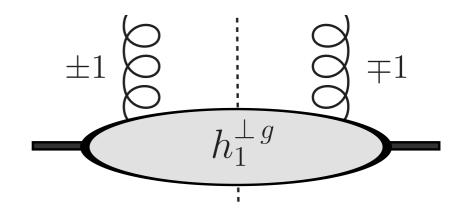
#### Gluon polarization inside unpolarized protons

Linearly polarized gluons can exist in unpolarized hadrons

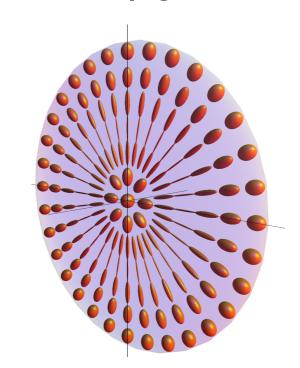
[Mulders, Rodrigues, 2001]



For  $h_1^{\perp g} > 0$  gluons prefer to be polarized along k<sub>T</sub>, with a  $\cos 2\phi$  distribution of linear polarization around it, where  $\phi = \angle(k_T, \epsilon_T)$ 



an interference between ±1 helicity gluon states



This TMD is  $k_T$ -even, chiral-even and T-even:

$$\Gamma_U^{\mu\nu}(x, \mathbf{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, \mathbf{p}_T^2) + \left( \frac{p_T^{\mu} p_T^{\nu}}{M_p^2} + g_T^{\mu\nu} \frac{\mathbf{p}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{p}_T^2) \right\}$$

#### Linear gluon polarization at small x

 $h_1^{\perp g}$  is more difficult to extract, as it cannot be probed in DIS, DY, SIDIS, nor in inclusive hadron or  $\gamma$ +jet production in pp or pA collisions

Selection of processes that probe the WW or DP linearly polarized gluon TMD:

	$pp \to \gamma \gamma X$	$pA \to \gamma^* \operatorname{jet} X$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		$\begin{array}{ c c c c c c c c c c c c c c c c c c c$
$h_1^{\perp g[+,+]} \text{ (WW)}$		×			
$h_1^{\perp g[+,-]} \text{ (DP)}$	×		×	×	×

Higgs and  $0^{\pm +}$  quarkonium production allows to measure the linear gluon polarization using the angular independent  $p_T$  distribution

All other suggestions use angular modulations

EIC and RHIC/LHC can probe same h<sub>1</sub><sup>-1g</sup>

Qiu, Schlegel, Vogelsang, 2011; Jian Zhou, 2016; D.B., Brodsky, Pisano, Mulders, 2011; D.B., Pisano, 2012; Sun, Xiao, Yuan, 2011; D.B., den Dunnen, Pisano, Schlegel, Vogelsang, 2012; den Dunnen, Lansberg, Piano, Schlegel, 2014

#### Linear gluon polarization at small x

There is no theoretical reason why  $h_1^{\perp g}$  effects should be small, especially at small x

Evolution:  $h_1^{\perp g}$  has the same 1/x growth as  $f_1$ 

$$\tilde{h}_{1}^{\perp g}(x, b^{2}; \mu_{b}^{2}, \mu_{b}) = \frac{\alpha_{s}(\mu_{b})C_{A}}{2\pi} \int_{x}^{1} \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{g/P}(\hat{x}; \mu_{b}) + \mathcal{O}(\alpha_{s}^{2})$$

The small-x limit of the DP correlator in the TMD formalism:

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \stackrel{x\to 0}{\longrightarrow} \frac{k_T^i k_T^j}{2\pi L} \Gamma_0^{[\Box]}(\boldsymbol{k}_T) \qquad U^{[\Box]} = U_{[0,y]}^{[+]} U_{[y,0]}^{[-]}$$

$$\Gamma_U^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[ -g_T^{ij} f_1(x, \mathbf{k}_T^2) + \frac{k_T^{ij}}{M^2} h_1^{\perp}(x, \mathbf{k}_T^2) \right] \xrightarrow{x \to 0} \frac{k_T^i k_T^j}{2M^2} e(\mathbf{k}_T^2)$$

$$\lim_{x \to 0} x f_1(x, \mathbf{k}_T^2) = \frac{\mathbf{k}_T^2}{2M^2} \lim_{x \to 0} x h_1^{\perp}(x, \mathbf{k}_T^2) = \frac{\mathbf{k}_T^2}{2M^2} e(\mathbf{k}_T^2)$$

The DP  $h_1^{\perp g}$  becomes maximal when  $x \to 0$ 

D.B., Cotogno, van Daal, Mulders, Signori, Zhou, 2016

#### Polarization of the CGC

CGC framework calculations show the CGC gluons are in fact linearly polarized

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g}$$
 for  $k_{\perp} \ll Q_s$ ,  $h_{1,WW}^{\perp g} = 2f_{1,WW}^{\perp g}$  for  $k_{\perp} \gg Q_s$ 

$$xh_{1,DP}^{\perp g}(x,k_{\perp}) = 2xf_{1,DP}^{g}(x,k_{\perp})$$

Metz, Zhou '11



$$\frac{h_{1\,WW}^{\perp\,g}}{f_{1\,WW}} \propto \frac{1}{\ln Q_s^2/k_\perp^2}$$



The "k<sub>T</sub>-factorization" approach (CCFM) yields maximum polarization too:

$$\Gamma_g^{\mu
u}(x,m{p}_T)_{ ext{max pol}} = rac{p_T^\mu p_T^
u}{m{p}_T^2}\,x\,f_1^g$$
 Catani, Ciafaloni, Hautmann, 1991

#### Dijet production at EIC

WW  $h_1^{\perp g}$  accessible in dijet production in eA collisions at a high-energy EIC

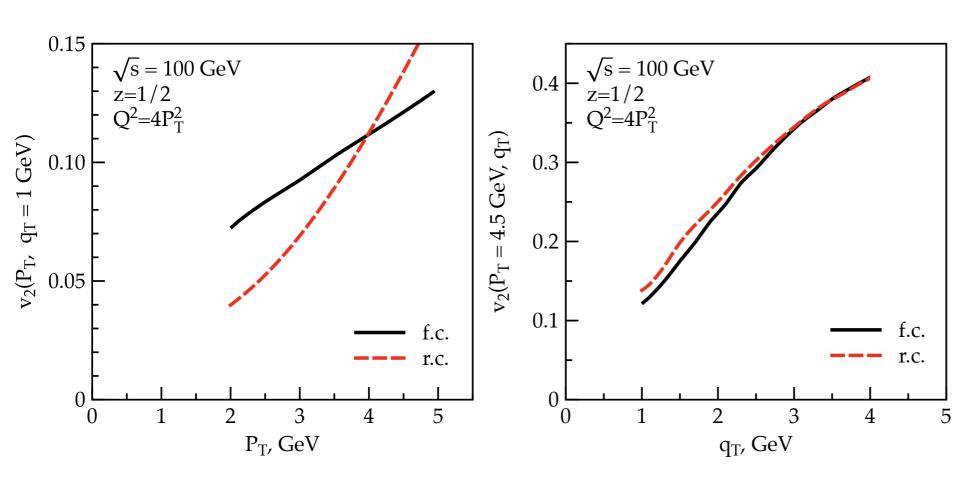
[Metz, Zhou 2011; Pisano, D.B., Brodsky, Buffing, Mulders, 2013; D.B., Pisano, Mulders, Zhou, 2016]

The WW  $h_1^{\perp g}$  is (moderately) suppressed for small transverse momenta:

$$\frac{h_{1\,WW}^{\perp\,g}}{f_{1\,WW}} \propto \frac{1}{\ln Q_s^2/k_\perp^2}$$

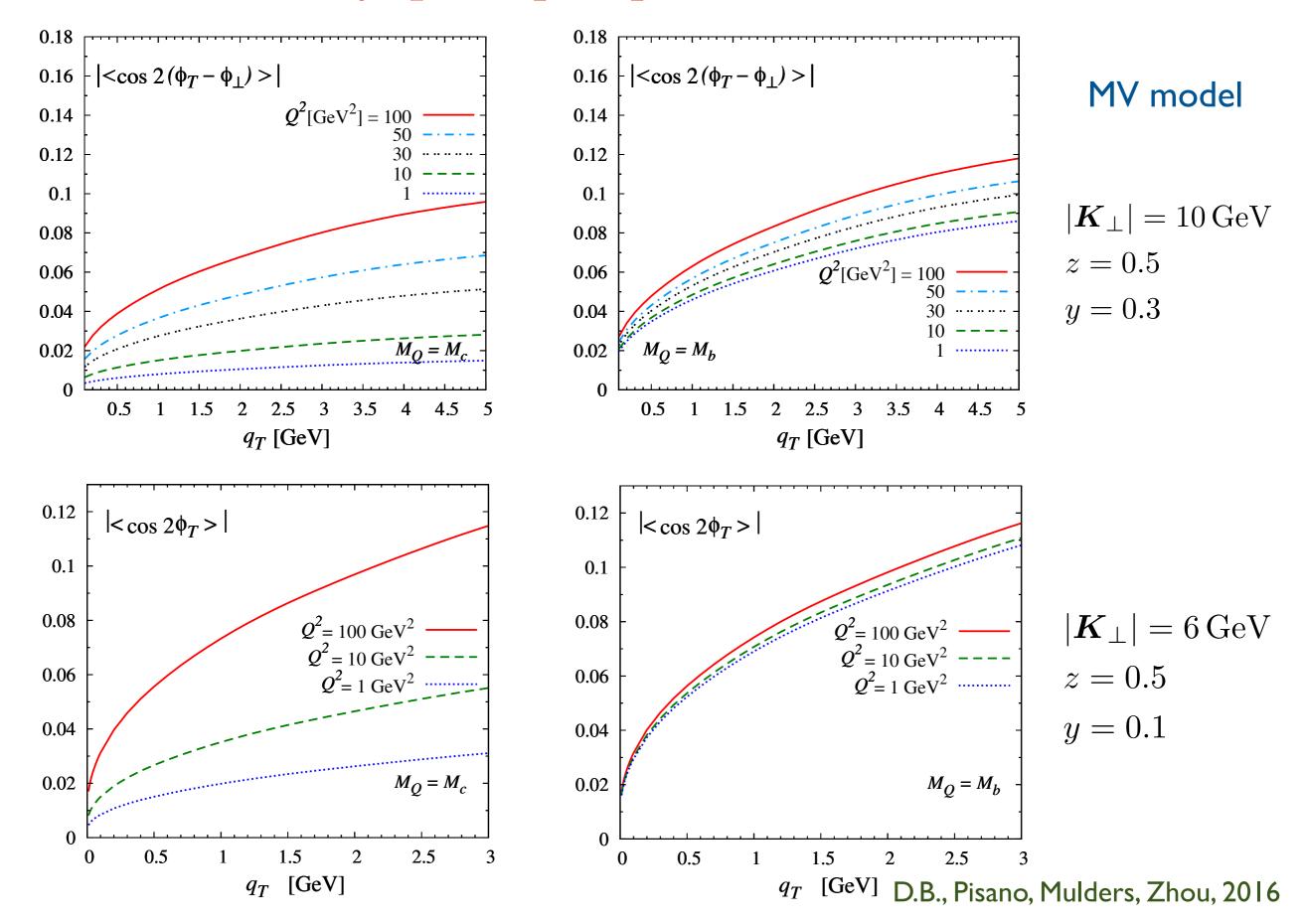
Metz, Zhou '11

Polarization shows itself through a cos2φ distribution



Large effects are found Dumitru, Lappi, Skokov, 2015

#### Heavy quark pair production at EIC



# Gluon Sivers effect at small x

#### Small gluon Sivers effect?

#### Arguments suggesting gluon Sivers is small:

- Burkardt sum rule already (approximately) satisfied by up and down quarks

$$\sum_{a=q,g} \int f_{1T}^{\perp(1)a}(x) \, dx = 0$$

- small Sivers asymmetry on deuteron target as found by COMPASS [Brodsky & Gardner, 2006]
- $I/N_c$  suppressed at not too small x (x~ $I/N_c$ ), of order of the flavor singlet u+d [Efremov, Goeke, Menzel, Metz, Schweitzer, 2005]
- small A<sub>N</sub> at midrapidity (small gluon Sivers function in the GPM) [Anselmino, D'Alesio, Melis & Murgia, 2006; D'Alesio, Murgia, Pisano, 2015]

Note however that  $A_N$  in pion production is not a TMD factorizing process COMPASS high-pT hadron pairs and other constraints are about fairly large x

Gluon Sivers function is constrained to be  $\leq 30\%$  of nonsinglet quark Sivers function. This is of natural size and will lead to smaller asymmetries, but not necessarily tiny

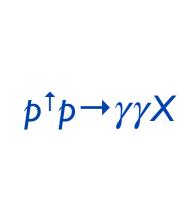
#### Gluon Sivers effect at small x

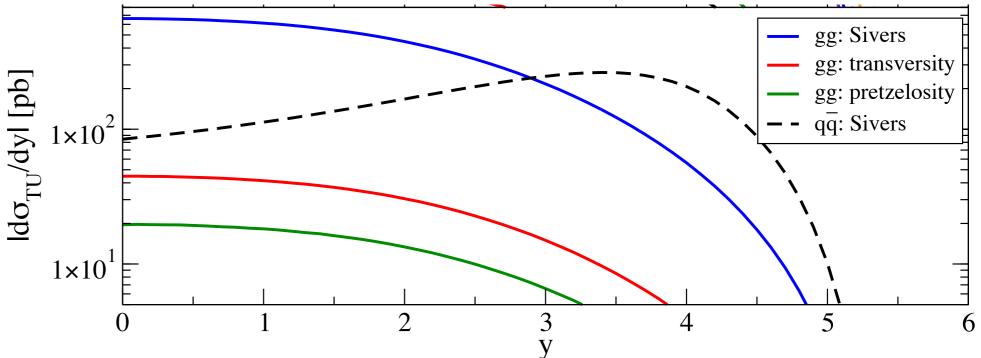
#### Selection of processes that probe the WW (f type) or DP (d type) Sivers gluon TMD:

	DY	SIDIS	$p^{\uparrow} A \to h X$	$p^{\uparrow}A \to \gamma^{(*)} \text{ jet } X$		$\begin{array}{c} e \ p^{\uparrow} \rightarrow e' \ Q \ \overline{Q} \ X \\ e \ p^{\uparrow} \rightarrow e' \ j_1 \ j_2 \ X \end{array}$
$f_{1T}^{\perp g[+,+]}(\mathrm{WW})$	×	×	×	×	$\sqrt{}$	
$f_{1T}^{\perp g[+,-]}(\mathrm{DP})$	$\sqrt{}$	$\sqrt{}$			×	×

backward hadron production

[Qiu, Schlegel, Vogelsang, 2011]





 $\sqrt{s}=500$  GeV,  $p_T^{\gamma} \ge 1$  GeV, integrated over  $4 < Q^2 < 30$  GeV<sup>2</sup>,  $0 \le q_T \le 1$  GeV

At photon pair rapidity y < 3 gluon Sivers dominates and max( $d\sigma_{TU}/d\sigma_{UU}$ ) ~ 30-50%

#### Gluon Sivers effect at small x

At small x the large  $k_T$  tail of the WW Sivers function is suppressed by a factor of x compared to the unpolarized gluon function

The DP-type Sivers function is not suppressed and can be probed in pA collisions

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \stackrel{x\to 0}{\longrightarrow} \frac{k_T^i k_T^j}{2\pi L} \Gamma_0^{[\square]}(\boldsymbol{k}_T)$$

The DP-type Sivers function at small x turns out to be the spin-dependent odderon

$$\Gamma_{(d)}^{(T-\text{odd})} \equiv \left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \text{F.T.} \langle P, S_T | \text{Tr} \left[U^{[\Box]}(0_T, y_T) - U^{[\Box]\dagger}(0_T, y_T)\right] | P, S_T \rangle$$

D.B., Echevarria, Mulders, Zhou, 2016

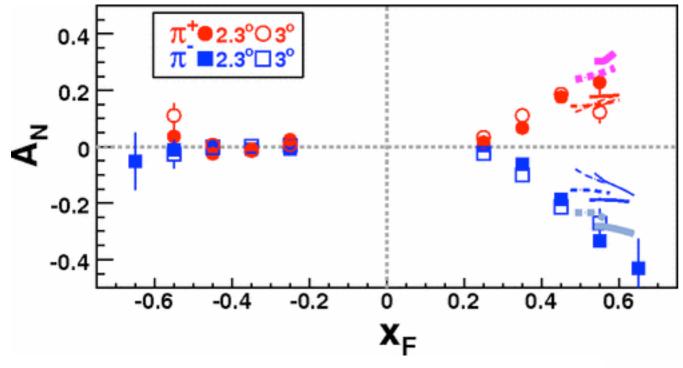
a single Wilson loop matrix element

$$U^{[\Box]} = U_{[0,y]}^{[+]} U_{[y,0]}^{[-]}$$

It is the only relevant contribution in  $A_N$  at negative  $x_F$ , as opposed to the many contributions at positive  $x_F$ 

The imaginary part of the Wilson loop determines the gluonic single spin asymmetry

#### $p^{\uparrow}p \rightarrow h^{\pm} X \text{ at } x_F < 0$

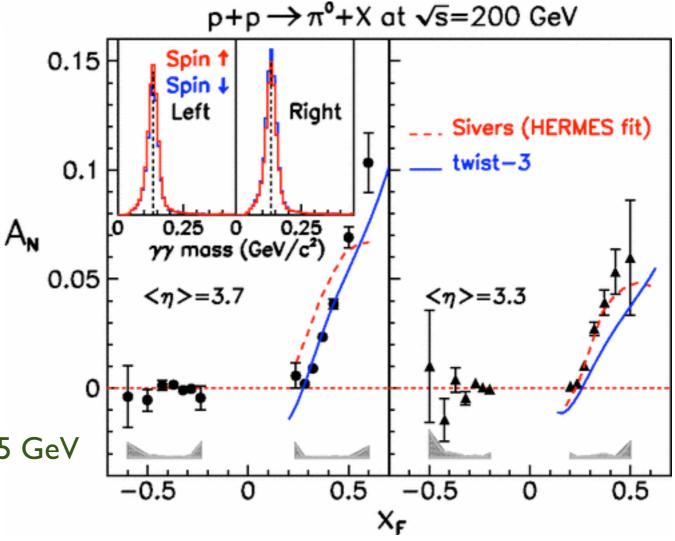


BRAHMS, 2008  $\sqrt{s} = 62.4 \text{ GeV}$ low p<sub>T</sub>, up to roughly 1.2 GeV where gg channel dominates

spin-dependent odderon is C-odd, whereas gg in the CS state is C-even

expect smaller asymmetries in neutral pion and jet production

STAR, 2008  $\sqrt{s} = 200 \text{ GeV}$  pt between I and 3.5 GeV



## Conclusions

#### Conclusions

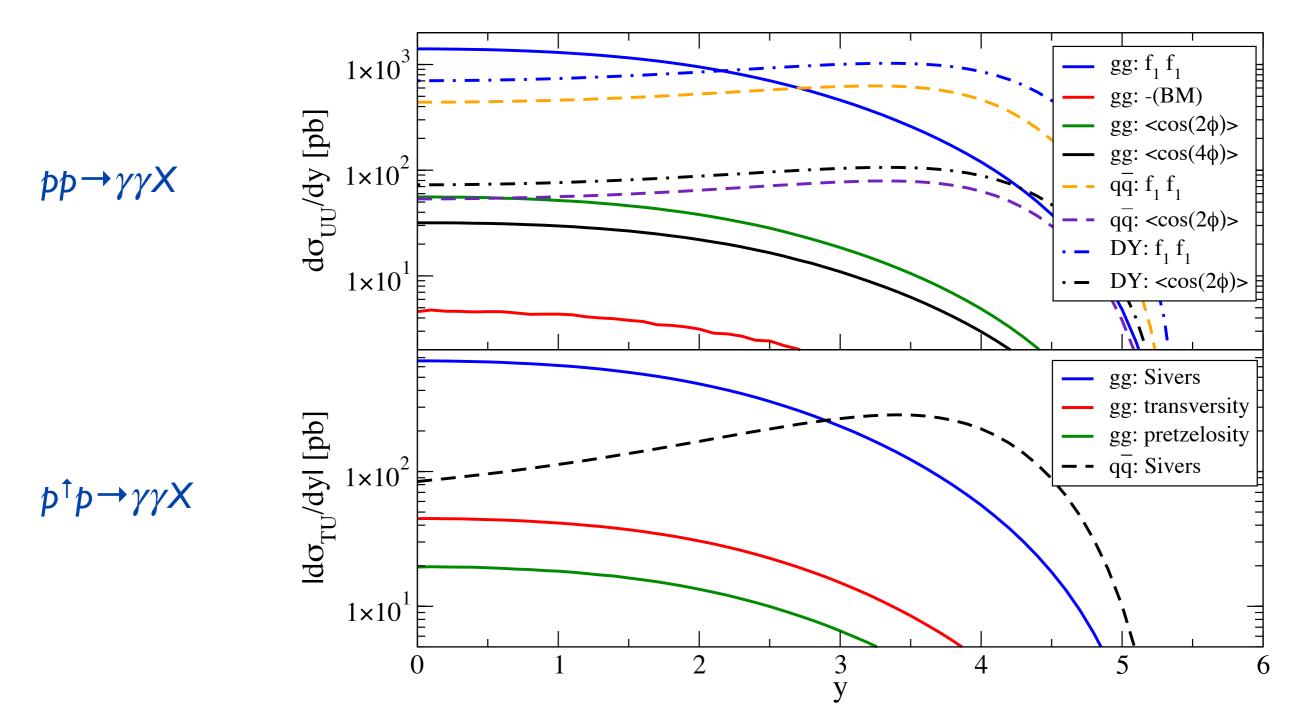
- All TMDs are process dependent, with observable and testable effects
- At small x the unpolarized WW and DP gluon TMDs both matter and there are sufficient processes in ep and pp collisions to test the expectations
- Same applies to the linear polarization of gluons inside unpolarized hadrons, which can lead to sizable effects for cos2φ asymmetries at EIC
- The CGC can be maximally polarized, although not all processes will be (fully) sensitive to it
- Two distinct gluon Sivers TMDs can be measured in p<sup>†</sup>p and p<sup>†</sup>A collisions
   (RHIC & AFTER@LHC), the WW-type allows for a sign-change test w.r.t. ep<sup>†</sup> (EIC)
- As  $x \rightarrow 0$  only the DP gluon Sivers TMD remains, which then corresponds to the spin-dependent odderon, a T-odd and C-odd single Wilson loop matrix element that determines  $A_N$  at negative  $x_F$

#### Still to be done:

studies of TMD factorization of  $\gamma^{(*)}$ +jet,  $J/\psi+\gamma$ ,  $J/\psi+J/\psi$  production in pp/pA collisions and of effective TMD factorization (hybrid factorization) at small x

# Back-up slides

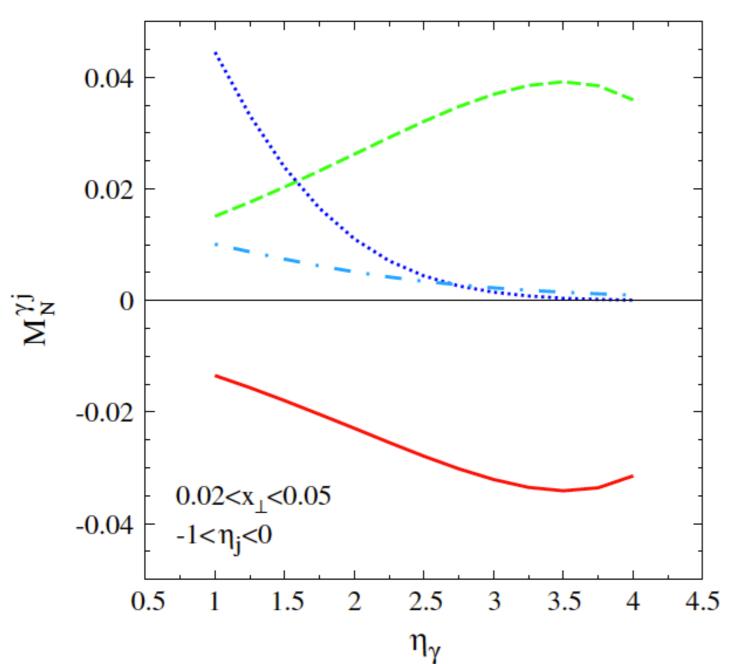
#### Photon pair production



 $\sqrt{s}$ =500 GeV,  $p_T^{\gamma} \ge I$  GeV, integrated over  $4 < Q^2 < 30$  GeV<sup>2</sup>,  $0 \le q_T \le I$  GeV At photon pair rapidity y < 3 gluon Sivers dominates and max( $d\sigma_{TU}/d\sigma_{UU}$ ) ~ 30-50%

#### Photon-jet production

$$M_N^{\gamma j}(\eta_{\gamma}, \eta_j, x_{\perp}) = \frac{\int d\phi_j \, d\phi_{\gamma} \frac{2|\mathbf{K}_{\gamma \perp}|}{M} \sin(\delta\phi) \cos(\phi_{\gamma}) \frac{d\sigma}{d\phi_j \, d\phi_{\gamma}}}{\int d\phi_j \, d\phi_{\gamma} \frac{d\sigma}{d\phi_j \, d\phi_{\gamma}}}$$



Prediction for the azimuthal moment at  $\sqrt{s}$ =200 GeV,  $p_T^{\gamma} \ge 1$  GeV, integrated over  $-1 \le \eta_j \le 0, 0.02 \le x_{\perp} \le 0.05$ 

Dashed line: GPM

Solid line: using gluonic-pole cross sections

Dotted line: maximum contribution from the gluon Sivers function (absolute value)

Dot-dashed line: maximum contribution from the Boer-Mulders function (abs. value)

[Bacchetta, Bomhof, D'Alesio, Mulders, Murgia, 2007]

#### WW vs DP

At small x the unpolarized WW and DP gluon TMDs both matter and there are sufficient processes in ep and pp collisions to test the expectations

How different can the two unpolarized gluon distributions be?

The first transverse moment must coincide

$$\int d\mathbf{k}_T f_1^{g\,[+,+]}(x,\mathbf{k}_T^2) = \int d\mathbf{k}_T f_1^{g\,[+,-]}(x,\mathbf{k}_T^2)$$

Also the large k<sub>T</sub> tail of the functions must coincide

Therefore, the two functions can have rather different shapes and magnitudes